

“What practitioners need to know about utility”

by Mark Kritzman, Financial Analysts Journal, May- June, 1992

Rational investors try to maximize the expected utility. The price of something may be the same for everyone but the utility would depend on the individual and her circumstances.

Utility increases with wealth but at a diminishing rate. Bernoulli's notion of utility implies that its first derivative with respect to wealth is positive while its second derivative is negative. We would also experience greater disutility from a decline in wealth than the utility we would derive from an equivalent increase in wealth.

According to Bernoulli, the change in utility equals the logarithm of the sum of the initial wealth plus the increment to the wealth, divided by the initial wealth.

When wealth increases from 100 to 150, increase in utility is given by:

$$\text{Ln}(150/100) = 0.4055$$

When wealth increases from 150 to 200, increase in utility is given by:

$$\text{Ln}(200/150) = 0.2877$$

Bernoulli's concept of utility implies that we prefer a certain outcome to an uncertain outcome of equal value.

We can extend Bernoulli's insight and determine how much value we would be willing to subtract from a certain prospect before we would select a risky prospect. The value of the certain prospect that yields the same utility as the expected utility of an uncertain prospect is called the certainty equivalent.

In many applications, it is convenient to model utility as a function of expected return and risk as measured by the standard deviation of returns. It is possible to identify combinations of expected return and standard deviation that yield the same level of utility for a particular risk-aversion coefficient. Drawing a curve through all the combinations of expected return and risk with equal utility creates an indifference curve.

The concept of utility is critical to choice under uncertainty. However, concise mathematical models of utility may not always fully reflect investor attitudes and idiosyncrasies. Thus, there are investors who are risk averse, those who are risk neutral and those who seek risk. A risk-averse person will reject a fair game, while a risk-neutral person will be indifferent to a fair game and a risk seeker will select a fair game.

The relationship between wealth and risk taking may also not fit into just one category. Decreasing absolute risk aversion indicates that the amount of wealth one is willing to expose to risk increases as one's wealth increases. Constant absolute risk aversion implies that the amount of wealth exposed to risk remains unchanged as wealth increases. Increasing absolute risk aversion means that absolute risk exposure decreases as wealth increases.

Relative risk aversion refers to changes in the percentage of one's wealth exposed to risk as wealth increases. Decreasing relative risk aversion implies that the percentage of wealth exposed to risk increases as wealth increases. With constant relative risk aversion, the percentage of wealth exposed to risk does not change as wealth increases. Increasing relative risk aversion implies that percentage risk exposure decreases as wealth increases.