What practitioners need to know about time diversification
by Mark Kritzman, Financial Analysts Journal, January-February 2015

The notion that above-average returns tend to offset below-average returns over long horizons is called time diversification. Specifically, if returns are independent from one year to the next, the standard deviation of annualized returns diminishes with time. The distribution of annualized returns consequently converges as the investment horizon increases.

One problem with time diversification is that while the annualized dispersion of returns converges toward the expected return with the passage of time, the dispersion of terminal wealth also diverges from the expected terminal wealth as the investment horizon expands. This result implies that, although we are less likely to lose money over a long horizon than over a short horizon, the magnitude of our potential loss increases with the duration of our investment horizon. Proponents of time diversification would counter that although the dispersion of terminal wealth increases with the passage of time, the expected wealth of the risky venture also increases. The dispersion of wealth thus expands around a growing mean as the horizon lengthens.

It is an indisputable mathematical fact that if we prefer a riskless asset to a risky asset given a three-month horizon, we should also prefer a riskless asset to a risky asset given a 10-year horizon, assuming the following conditions are satisfied:

- Our risk aversion is invariant to changes in our wealth.
- We believe that risky asset returns are random.
- Our future wealth depends only on investment results.

Despite the mathematical proof, we may still subscribe to time diversification because one or more of the above conditions may not be fulfilled.

We may not believe that risky asset returns are random. Perhaps investment returns follow a mean-reverting pattern. In that case, the dispersion of terminal wealth increases at a slower rate than implied by a lognormal distribution (the distribution that results from random returns). If we are more averse to risk than the degree of risk aversion implicit in a log wealth utility function, then a mean-reverting process will lead us to favour risky assets over a long horizon, even if we are indifferent between a riskless and a risky asset over a short horizon.

We might believe that the extremely bad outcomes required to justify the irrelevancy of time diversification would result from events or conditions that would have equally dire consequences for the so-called riskless asset, especially if we measure wealth in consumption units.

Even if we believe that returns are random, we might still choose to accept more risk over longer horizons than over shorter horizons because we have more discretion to adjust our consumption and work habits.

We may have a discontinuous utility function. Suppose we require a minimum level of wealth to maintain a certain standard of living. Our lifestyle might change drastically if we penetrate this threshold, but further reductions in wealth are less meaningful. We might be more likely to penetrate the threshold given a risky investment over a short horizon than we would be, if we invested in the same risky asset over the long run.