"What practitioners need to know about log normality"

by Mark Kritzman, Financial Analysts Journal, July-August, 1992

A normal distribution is symmetric and is fully described by the mean and the variance. Higher moments such as skewness have a value of zero.

A lognormal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed.

We have to compound the returns generated in smaller periods of time to arrive at the return for the whole period. We can take logarithms on both sides. The natural log of the quantity 1 plus the periodic rate of return equals the corresponding continuous rate of return.

Periodic returns= $(P_2 – P_1) / P_1$
When we use continuous compounding,
$P_2 = P_1 e^r$
$Ln (P_2 / P_1) = ln (e^r) = r$
$Ln [(P_2 - P_1) / P_1 + 1] = r$

Because we sum logarithms, the natural logs of the quantities 1 plus the periodic returns are normally distributed. Because these natural logs are normally distributed, and because the exponential of the normal distribution gives the lognormal distribution, the quantities 1 plus the periodic returns, which are the exponentials of the natural logs, are lognormally distributed.

Given a one-year time horizon, it may not make much of a difference whether we assume a normal or a log normal distribution to estimate probabilities. As we extend the time horizon, the normal distribution overestimates the probability of achieving the target returns that are below the expected returns and underestimates the probability of achieving target returns above the expected returns.