What practitioners need to know about duration and convexity

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Duration and convexity are indispensable tools for fixed income investors.

A bond's maturity is the time to receipt of the final principal repayment. Macaulay's Duration is the average time to receipt of a bond's cash flows in which each cash flow's time to receipt is weighted by its present value as percentage of the total present value of all the cash flows. Of course, the total present value of all the cash flows equals the bond price. When Macaulay's duration is divided by one plus yield to maturity, we get modified duration. Multiplying the change in yield by modified duration tells us the percentage change in bond price.

Convexity is the extent to which duration changes as interest rates change. The larger the increase in yield to maturity, the greater the magnitude of the error by which the modified duration will overestimate the bond's price decline. The larger the decrease in in yield to maturity, the greater the magnitude of the error by which the modified duration will underestimate the bond's price increase.

Convexity arises because when yield to maturity changes, the duration changes as well. So, the price change of the bond depends not only on the duration but also on the convexity. Convexity is more pronounced the farther apart the cash flows are.

Duration enables portfolio managers to act upon their convictions about interest rate shifts. An investment manager who expects interest rates to fall, should increase the duration of the portfolio to maximize gains. If the manager expects the rate to increase, she should reduce the duration to minimize losses.

A portfolio manager can hedge a liability stream by constructing a portfolio of equal duration and convexity, as long as its present value equals the present value of the liabilities at the outset.

If the present value of the liabilities exceeds the present value of the assets available for hedging, the duration of the portfolio must exceed the duration of the liabilities. The converse is true if the value of the portfolio exceeds the value of the liabilities.

A portfolio can be immunized from interest rate shifts by setting its duration equal to the investor's holding period. If interest rates rise, the capital loss

will be offset by the gain from reinvesting the cash flows at higher yields. Conversely, if interest rates fall, the reduction in income due to reinvestments at lower rates is offset by the capital gain. Of course, capital gains and losses are balanced by reinvestment gains and losses only if short-term rates and long-term rates move together. If long-term rates increase but short-term rates remain unchanged, the portfolio's income will not increase sufficiently to offset the capital loss. Then immunization will fail.