Credit Risk Management: A Primer

By A. V. Vedpuriswar
Altman’s Z Score

- Altman’s Z score is a good example of a credit scoring tool based on data available in financial statements.
- It is an empirical model based on multiple discriminant analysis.
- The Z score is calculated as:
  \[ Z = 1.2x_1 + 1.4x_2 + 3.3x_3 + 0.6x_4 + 0.999x_5 \]
  - \( x_1 \) = Working capital / Total assets
  - \( x_2 \) = Retained earnings / Total assets
  - \( x_3 \) = Earnings before interest and taxes / Total assets
  - \( x_4 \) = Market value of equity / Book value of total liabilities
  - \( x_5 \) = Sales / Total assets

Altman’s Z Score

- Companies with low Z-scores are more likely to default than companies with high Z-scores.
- Altman used statistical techniques to determine the best weights to put on each ratio.
- The most significant financial ratio for predicting default is earnings before income and taxes divided by total assets.
- The next most significant financial ratio is sales to total assets.
- Altman’s model does not take into account that the characteristics (e.g., financial ratios) of companies change over time.
Interpreting Z scores

Once Z is calculated, the credit risk is assessed as follows:

- Z > 3.0 means low probability of default
- 2.7 < Z < 3.0 means an alert signal
- 1.8 < Z < 2.7 means a good chance of default
- Z < 1.8 means a high probability of default
Shumway’s hazard rate model (1)

- Shumway (2001) estimated a hazard rate model of default.

- Hazard rate models are widely used in the insurance industry to estimate the probability that a risk event will happen in time, $t$.

- If $\lambda^*$ is the hazard rate for an event (e.g., default), then $1 - e^{(-\lambda t)}$ is the probability that the event will occur at or before time $t$.

- For small $t$, this is approximately equal to $\lambda t$.

- Thus the probability of default over a short time period is the hazard rate for default multiplied by the length of the time period.

- The hazard rate depends on current financial ratios as well as market capitalization, excess equity return, and equity-return volatility.

- The inclusion of market-driven variables improves predictive ability.
Shumway’s hazard rate model (2)

♦ In Shumway’s model, the hazard rate depended on current financial ratios as well as market variables like market capitalization, excess equity return, and equity-return volatility.

♦ The inclusion of these market-driven variables improves the predictive ability of hazard rate models.

♦ Shumway found that the only financial ratios with predictive power are EBIT/total liabilities and Market equity/Total liabilities.
Credit risk may appear in Banking and trading books

♦ The banking book covers credit risk arising from:
  – commercial loans
  – loans to sovereigns and public sector entities
  – consumer (retail) loans

♦ Some financial instruments that give rise to credit risk do not appear on a bank's books.

♦ These off-balance sheet items include loan commitments and lines of credit.

♦ They may be converted later to on-balance sheet items.

♦ The trading book covers credit risk arising from
  – exchange traded instruments and
  – OTC derivatives.
The Building blocks of Credit Risk Management

- **Probability of default**
  - Refers to the likelihood of borrower not honoring contractual obligations; is a measure of the expected default frequency.

- **Exposure at default**
  - Maximum amount an institution can lose if a borrower or counterparty defaults.

- **Loss given default**
  - The percentage of an outstanding claim that cannot be recovered in the event of a default.
### Banking and traded products exposure by business division and Corporate Center unit

<table>
<thead>
<tr>
<th>CHF million</th>
<th>Wealth Management</th>
<th>Wealth Management Americas</th>
<th>Personal &amp; Corporate Banking</th>
<th>Asset Management</th>
<th>Investment Bank</th>
<th>CC – Services</th>
<th>CC – Group</th>
<th>CC – ALM</th>
<th>CC – Non-core and Legacy Portfolio</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balances with central banks</td>
<td>427</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>32</td>
<td>0</td>
<td>86,618</td>
<td>0</td>
<td>0</td>
<td>87,078</td>
</tr>
<tr>
<td>Due from banks</td>
<td>1,356</td>
<td>3,357</td>
<td>1,485</td>
<td>570</td>
<td>8,725</td>
<td>356</td>
<td>2,740</td>
<td>0</td>
<td>0</td>
<td>18,589</td>
</tr>
<tr>
<td>Loans¹</td>
<td>115,180</td>
<td>53,014</td>
<td>131,380</td>
<td>1</td>
<td>12,094</td>
<td>34</td>
<td>7,226</td>
<td>88</td>
<td>319,016</td>
<td></td>
</tr>
<tr>
<td>Guarantees</td>
<td>1,982</td>
<td>460</td>
<td>9,551</td>
<td>0</td>
<td>5,040</td>
<td>105</td>
<td>2</td>
<td>2</td>
<td>17,142</td>
<td></td>
</tr>
<tr>
<td>Loan commitments</td>
<td>1,861</td>
<td>347</td>
<td>9,160</td>
<td>0</td>
<td>20,619</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>31,988</td>
<td></td>
</tr>
<tr>
<td>Banking products exposure²</td>
<td>120,806</td>
<td>57,178</td>
<td>151,576</td>
<td>570</td>
<td>46,510</td>
<td>496</td>
<td>96,585</td>
<td>90</td>
<td>473,813³</td>
<td></td>
</tr>
<tr>
<td>Banking products exposure, net⁴</td>
<td>120,701</td>
<td>57,153</td>
<td>151,105</td>
<td>570</td>
<td>44,693</td>
<td>496</td>
<td>96,585</td>
<td>61</td>
<td>471,364</td>
<td></td>
</tr>
<tr>
<td>Over-the-counter derivatives⁵</td>
<td>5,547</td>
<td>26</td>
<td>1,234</td>
<td>0</td>
<td></td>
<td>11,444</td>
<td></td>
<td></td>
<td>18,250</td>
<td></td>
</tr>
<tr>
<td>Securities financing transactions⁵</td>
<td>0</td>
<td>222</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td>17,842</td>
<td></td>
<td>18,064</td>
<td></td>
</tr>
<tr>
<td>Exchange-traded derivatives⁵</td>
<td>963</td>
<td>1,730</td>
<td>76</td>
<td>0</td>
<td></td>
<td>5,444</td>
<td></td>
<td></td>
<td>8,213</td>
<td></td>
</tr>
<tr>
<td>Traded products exposure⁵</td>
<td>6,510</td>
<td>1,978</td>
<td>1,310</td>
<td>0</td>
<td></td>
<td>34,729</td>
<td></td>
<td></td>
<td>44,527</td>
<td></td>
</tr>
<tr>
<td>Traded products exposure, net⁵</td>
<td>6,510</td>
<td>1,978</td>
<td>1,310</td>
<td>0</td>
<td></td>
<td>33,996</td>
<td></td>
<td></td>
<td>43,794</td>
<td></td>
</tr>
<tr>
<td>Credit exposure⁵</td>
<td>127,316</td>
<td>59,156</td>
<td>152,886</td>
<td>570</td>
<td></td>
<td>178,411</td>
<td></td>
<td></td>
<td>518,339</td>
<td></td>
</tr>
<tr>
<td>Credit exposure, net⁶</td>
<td>127,211</td>
<td>59,131</td>
<td>152,414</td>
<td>570</td>
<td></td>
<td>175,832</td>
<td></td>
<td></td>
<td>515,158</td>
<td></td>
</tr>
</tbody>
</table>
### UBS: Distribution of exposures

#### Personal & Corporate Banking: distribution of banking products exposure across internal UBS ratings and loss given default (LGD) buckets

<table>
<thead>
<tr>
<th>CHF million, except where indicated</th>
<th>31.12.17</th>
<th>31.12.16</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LGD buckets</strong></td>
<td>Exposure</td>
<td>0–25%</td>
</tr>
<tr>
<td>Investment grade</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment grade</td>
<td>89,975</td>
<td>53,566</td>
</tr>
<tr>
<td>Sub-investment grade</td>
<td>61,602</td>
<td>27,210</td>
</tr>
<tr>
<td>of which: 6–9</td>
<td>55,730</td>
<td>25,234</td>
</tr>
<tr>
<td>of which: 10–12</td>
<td>4,040</td>
<td>1,894</td>
</tr>
<tr>
<td>of which: 13 and defaulted</td>
<td>1,832</td>
<td>82</td>
</tr>
<tr>
<td>Total exposure before deduction of allowances and provisions</td>
<td>151,576</td>
<td>80,776</td>
</tr>
<tr>
<td>Less: allowances and provisions</td>
<td>(472)</td>
<td></td>
</tr>
<tr>
<td>Net banking products exposure</td>
<td>151,105</td>
<td></td>
</tr>
</tbody>
</table>

1 The ratings of the major credit rating agencies, and their mapping to our internal rating scale, are shown in the “Internal UBS rating scale and mapping of external ratings” table in this section.
### Personal & Corporate Banking: unsecured loans by industry sector

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>31.12.17</th>
<th></th>
<th>31.12.16</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CHF million</td>
<td>%</td>
<td>CHF million</td>
<td>%</td>
</tr>
<tr>
<td>Construction</td>
<td>127</td>
<td>1.3</td>
<td>140</td>
<td>1.5</td>
</tr>
<tr>
<td>Financial institutions</td>
<td>1,162</td>
<td>12.1</td>
<td>1,675</td>
<td>17.6</td>
</tr>
<tr>
<td>Hotels and restaurants</td>
<td>83</td>
<td>0.9</td>
<td>96</td>
<td>1.0</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1,779</td>
<td>18.5</td>
<td>1,188</td>
<td>12.5</td>
</tr>
<tr>
<td>Private households</td>
<td>1,367</td>
<td>14.2</td>
<td>1,334</td>
<td>14.0</td>
</tr>
<tr>
<td>Public authorities</td>
<td>877</td>
<td>9.1</td>
<td>1,221</td>
<td>12.9</td>
</tr>
<tr>
<td>Real estate and rentals</td>
<td>181</td>
<td>1.9</td>
<td>143</td>
<td>1.5</td>
</tr>
<tr>
<td>Retail and wholesale</td>
<td>1,978</td>
<td>20.6</td>
<td>1,694</td>
<td>17.8</td>
</tr>
<tr>
<td>Services</td>
<td>1,821</td>
<td>18.9</td>
<td>1,748</td>
<td>18.4</td>
</tr>
<tr>
<td>Other</td>
<td>236</td>
<td>2.5</td>
<td>258</td>
<td>2.7</td>
</tr>
<tr>
<td>Net exposure</td>
<td>9,611</td>
<td>100.0</td>
<td>9,496</td>
<td>100.0</td>
</tr>
</tbody>
</table>

UBS: Unsecured loans
# UBS: Credit ratings

### Internal UBS rating scale and mapping of external ratings

<table>
<thead>
<tr>
<th>Internal UBS rating</th>
<th>1-year PD range in %</th>
<th>Description</th>
<th>Moody's Investors Service mapping</th>
<th>Standard &amp; Poor's mapping</th>
<th>Fitch mapping</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 and 1</td>
<td>0.00–0.02</td>
<td>Investment grade</td>
<td>Aaa</td>
<td>AAA</td>
<td>AAA</td>
</tr>
<tr>
<td>2</td>
<td>0.02–0.05</td>
<td>Aa1 to Aa3</td>
<td>AA+ to AA–</td>
<td>AA+ to AA–</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.05–0.12</td>
<td>A1 to A3</td>
<td>A+ to A–</td>
<td>A+ to A–</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.12–0.25</td>
<td>Baa1 to Baa2</td>
<td>BBB+ to BBB</td>
<td>BBB+ to BBB</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.25–0.50</td>
<td>Baa3</td>
<td>BBB–</td>
<td>BBB–</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.50–0.80</td>
<td>Sub-investment grade</td>
<td>Ba1</td>
<td>BB+</td>
<td>BB+</td>
</tr>
<tr>
<td>7</td>
<td>0.80–1.30</td>
<td>Ba2</td>
<td>BB</td>
<td>BB</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.30–2.10</td>
<td>Ba3</td>
<td>BB–</td>
<td>BB–</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>2.10–3.50</td>
<td>B1</td>
<td>B+</td>
<td>B+</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3.50–6.00</td>
<td>B2</td>
<td>B</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>6.00–10.00</td>
<td>B3</td>
<td>B–</td>
<td>B–</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>10.00–17.00</td>
<td>Caa</td>
<td>CCC</td>
<td>CCC</td>
<td>CCC</td>
</tr>
<tr>
<td>13</td>
<td>&gt;17</td>
<td>Ca to C</td>
<td>CC to C</td>
<td>CC to C</td>
<td></td>
</tr>
</tbody>
</table>

**Counterparty is in default**

<table>
<thead>
<tr>
<th></th>
<th>Default</th>
<th>Defaulted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>D</td>
</tr>
</tbody>
</table>
Exposure at Default: Diffusion and amortisation

- There are two main effects that determine the credit exposure over time for a single transaction or for a portfolio of transactions with the same counterparty:
  - Diffusion
  - Amortization.

- As time passes, the “diffusion effect” tends to increase the exposure.

- There is greater variability and, hence, greater potential for market price factors to move significantly away from current levels.

- The “amortization effect” in contrast, tends to decrease the exposure over time.

- This is because it reduces the remaining cash flows that are exposed to default.

- For single cash flow products, such as FX forwards, the potential exposure peaks at the maturity of the transaction, because it is driven purely by diffusion effect.

- For products with multiple cash flows, such as interest-rate swaps, the potential exposure usually peaks at one-third to one-half of the way into the life of the transaction.
Arriving at counterparty exposure

There are three main components in calculating the distribution of counterparty-level credit exposure:

- Scenario generation
- Instrument valuation
- Portfolio aggregation
Scenario generation

- There are two ways that we can generate possible future values of the price factors.

- The first is to generate a “path” of the market factors through time, so that each simulation describes a possible trajectory from time \( t=0 \) to the longest simulation date, \( t=T \).

- The other method is to simulate directly from time \( t=0 \) to the relevant simulation date \( t \).
Instrument valuation

- The second step in credit exposure calculation is to value the instrument at different future times using the simulated scenarios.

- The valuation models used to calculate exposure could be very different from the front-office pricing models.

- Typically, analytical approximations or simplified valuation models are used.

- The front office can afford to spend several minutes or even hours for a trade valuation.

- But valuations in the credit exposure framework must be done much faster, because each instrument in the portfolio must be valued at many simulation dates for a few thousand market risk scenarios.

- Therefore, valuation models such as those that involve Monte Carlo simulations or numerical solutions of partial differential equations do not satisfy the requirements on computation time.
Loss Given Default

- LGD is the percentage of the credit exposure that the lender will lose if the borrower defaults.
- It is also referred to as loss 'severity'.
- The recovery rate is the percentage of the exposure that is recovered when an obligor defaults.
- The higher the recovery rate, the lower the LGD.
- LGD is better represented by a distribution than by a single figure.
- There is uncertainty about recovery both due to quantifiable as well as fuzzy factors like bargaining power of debtors, creditors.
Measuring recovery

- There are two commonly used measures of recovery.
  - The ultimate recovery
    - Measurement is difficult.
    - Only way out in case of illiquid bank loans.
  - The price of debt just after default
    - Measurement is easy provided the debt is traded.
    - Often applicable to bonds.
Factors affecting LGD

- Seniority
- Collateral
- Type of borrower/obligation
- Industry
- Jurisdiction
Seniority

- Seniority is certainly one of the key determinants of the level of recovery.

- Another concept introduced by Keisman and Van de Castle is debt cushion.

- The more debt is junior to a given bond, higher the recovery rate.

- According to their study, when the debt cushion is 75% or more, 89% of the loans have a present value of recoveries of over 90%.

- When the debt cushion is under 20%, 40% of the loans show a present value of recoveries of under 60%.

- But this argument does not hold when there are multiple commitments to different creditors with the same seniority.
Collateral

- Collateral is useful but should not lead to complacency.
- From a regulatory standpoint, it may have an adverse impact on bank monitoring.
- For lenders, the value of the collateral may drop when there is an economic downturn and more firms start defaulting.
- Collateral does not guarantee full recovery.
Industry

- Assets that can be readily used by other parties have higher liquidation values.

- Firms in some industries have large quantities of real estate that can be sold in the market.

- Other sectors may be more labour intensive.

- Some industries are plagued by structural problems and hence are not competitive.

- More competitive industries are associated with higher recovery.
Jurisdiction

- Bankruptcy proceedings in the UK and US take less time.
- In Continental Europe it can take much longer.
- In India, the proceedings can take even more time!
Correlation between PD and LGD

- PD and LGD are influenced to some extent by the same macroeconomic variables.
- As an economy enters a period of recession, default rates increase.
- This leads to a large quantity of assets being liquidated at a time when demand and consequently prices are low.
- So recovery rates also tend to be low.
Problem

♦ There are 10 bonds in a portfolio. The probability of default for each of the bonds over the coming year is 5%. These probabilities are independent of each other. What is the probability that exactly one bond defaults?

♦ Solution

♦ Required probability

\[ = (10)(.05)(.95)^9 \]

\[ = .3151 \]

\[ = 31.51\% \]
Problem

A Credit Default Swap (CDS) portfolio consists of 5 bonds with zero default correlation. One year default probabilities are 1%, 2%, 5%, 10% and 15% respectively. What is the probability that the protection seller will not have to pay compensation?

Solution

Probability of no default

= (.99)(.98)(.95)(.90)(.85)

= .7051

= 70.51%
Problem

♦ If the probability of default is 6% in year 1 and 8% in year 2, what is the cumulative probability of default during the two years? Assume default does not lead to bankruptcy.

♦ Solution

♦ Probability of default not happening in both years

\[ (.94) (.92) = .8648 \]

♦ Required probability = 1 - .8648 = .1352

♦ = 13.52%
Problem

- The 5 year cumulative probability of default for a bond is 15%. The marginal probability of default for the sixth year is 10%. What is the six year cumulative probability of default?

Solution

- Required probability

\[ = 1 - (.85)(.90) \]

\[ = .235 \]

\[ = 23.5\% \]
Calculating probability of default from bond yields

- How can we do this?
- What is the significance of yield?
Problem

- Calculate the implied probability of default if the one year T Bill yield is 9% and a 1 year zero coupon corporate bond is fetching 15.5%. Assume no amount can be recovered in case of default.

- Let the probability of default be \( p \)

- Returns from corporate bond = \( 1.155 \, (1-p) + (0) \, (p) \)

- Returns from treasury = 1.09.

- To prevent arbitrage,
  
  \[ 1.155(1-p) = 1.09 \]

  \[ p = 1 - \frac{1.09}{1.155} = 1 - .9437 \]

- Probability of default = \( .0563 = 5.63\% \)

- In the earlier problem, if the recovery is 80% in the case of a default, what is the default probability?

  \[ 1.155(1-p) + (.80) \times (1.155) \times (p) = 1.09 \]

  \[ .231p = 0.065 \]

  \[ p = 0.2814 \]
Problem

- The T Bill yield is 2.9% and the corporate bond yield is 5.6%. Assuming zero recovery, what is the implied probability of default?

Solution

- $1.029 = (1-p)(1.056)$
- Or $p = 2.56\%$
Problem

✿ A loan of $10 million is made to a counterparty with probability of default 2% and recovery rate of 40%. If the cost of funds is LIBOR, what should be the price of the loan?

✿ Solution

✿ \[.02 = \frac{\text{spread}}{[1-0.4]}\]

✿ Spread = \(.02 \times 0.6 = 0.012 = 1.2 \% = 120 \text{ basis points}\)

✿ So quote will be LIBOR + 120 bp.
Problem

- If 1 year and 2 year T Bills are fetching 11% and 12% and 1 year and 2 year corporate bonds are yielding 16.5% and 17%, what is the marginal probability of default for the corporate bond in the second year? Assume the recovery is zero.

- Yield during the 2\textsuperscript{nd} year can be worked out as follows:

  - Corporate bonds: \((1.165)(1+i) = 1.17^2\)
  - \(i = 17.5\%\)
  - Treasury: \((1.11)(1+i) = (1.12)^2\)
  - \(i = 13.00\%\)
  - \((1-p)(1.175) + (p)(0) = 1.13\)
  - \(p = 1 - .9617\)
  - Default probability = 3.83%
Problem

- The spread between the yield on a 3 year corporate bond and the yield on a similar risk free bond is 50 basis points. The recovery rate is 30%. What is the cumulative probability of default over the three year period?

- Spread = (Probability of default) (loss given default)
- or .005 = (p) (1-.3)
- or p = .005/0.7 = .00714
  = .71% per year
- No default over 3 years = (.9929) (.9929) (.9929)
  = .9789
- So cumulative probability of default = 1 – .9789 = .0211=2.11%
Problem

The spread between the yield on a 5 year bond and that on a similar risk free bond is 80 basis points. If the loss given default is 60%, estimate the average probability of default over the 5 year period. If the spread is 70 basis points for a 3 year bond, what is the probability of default over years 4, 5?

Probability of default over the 5 year period = $0.008/0.6 = 0.133$

Probability of default over the 3 year period = $0.007/0.6 = 0.1167$

$(1 - 0.133)^5 = (1 - 0.1167)^3 (1 - p)^2$

or $(1 - p)^2 = 0.9352/0.9654 = 0.9688$

or $1 - p = 0.9842$

or $p = 0.0158 = 1.58\%$
Real World vs Risk Neutral Default Probabilities, 7 year averages (Table 19.5, page 415)

<table>
<thead>
<tr>
<th>Rating</th>
<th>Historical Hazard Rate (% per annum)</th>
<th>Hazard Rate from bonds (% per annum)</th>
<th>Ratio</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>0.034</td>
<td>0.596</td>
<td>17.3</td>
<td>0.561</td>
</tr>
<tr>
<td>Aa</td>
<td>0.098</td>
<td>0.728</td>
<td>7.4</td>
<td>0.630</td>
</tr>
<tr>
<td>A</td>
<td>0.233</td>
<td>1.145</td>
<td>5.8</td>
<td>0.912</td>
</tr>
<tr>
<td>Baa</td>
<td>0.416</td>
<td>2.126</td>
<td>5.1</td>
<td>1.709</td>
</tr>
<tr>
<td>Ba</td>
<td>2.140</td>
<td>4.671</td>
<td>2.2</td>
<td>2.531</td>
</tr>
<tr>
<td>B</td>
<td>5.462</td>
<td>8.017</td>
<td>1.5</td>
<td>2.555</td>
</tr>
<tr>
<td>Caa</td>
<td>12.016</td>
<td>18.395</td>
<td>1.5</td>
<td>6.379</td>
</tr>
</tbody>
</table>

Ref: Risk Management and Financial Institutions 4e, Chapter 19, Copyright © John C. Hull 2015
Which probability should we use?

- We should use risk-neutral estimates for valuing credit derivatives and estimating the present value of the cost of default.
- We should use real world estimates for calculating credit VaR and scenario analysis.
Problem

♦ A four year corporate bond provides a 4% semi annual coupon and yields 5% while the risk free bond, of maturity 4 years, also with 4% semi annual coupon yields 3% with continuous compounding. The bonds are redeemable at maturity at a face value of 100.

♦ Defaults may take place at the end of each year.

♦ In case of default, the recovery rate is flat 30% of the face value.

♦ What is the risk neutral default probability?

Ref: John C Hull, Risk Management and Financial institutions
Solution (1)

- **Risk free bond**

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>PV factor $e^{-0.03t}$</th>
<th>PV</th>
<th>PV factor $e^{-0.05t}$</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2</td>
<td>0.9851</td>
<td>1.9702</td>
<td>0.9754</td>
<td>1.9508</td>
</tr>
<tr>
<td>1.0</td>
<td>2</td>
<td>0.9704</td>
<td>1.9408</td>
<td>0.9512</td>
<td>1.9024</td>
</tr>
<tr>
<td>1.5</td>
<td>2</td>
<td>0.9560</td>
<td>1.9120</td>
<td>0.9277</td>
<td>1.8554</td>
</tr>
<tr>
<td>2.0</td>
<td>2</td>
<td>0.9418</td>
<td>1.8836</td>
<td>0.9048</td>
<td>1.8096</td>
</tr>
<tr>
<td>2.5</td>
<td>2</td>
<td>0.9277</td>
<td>1.8554</td>
<td>0.8825</td>
<td>1.7650</td>
</tr>
<tr>
<td>3.0</td>
<td>2</td>
<td>0.9139</td>
<td>1.8278</td>
<td>0.8607</td>
<td>1.7214</td>
</tr>
<tr>
<td>3.5</td>
<td>2</td>
<td>0.9003</td>
<td>1.8006</td>
<td>0.8395</td>
<td>1.679</td>
</tr>
<tr>
<td>4.0</td>
<td>102</td>
<td>0.8869</td>
<td>90.4638</td>
<td>0.8187</td>
<td>83.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><strong>103.65</strong></td>
<td></td>
<td><strong>96.21</strong></td>
</tr>
</tbody>
</table>

- So expected value of losses = 103.65 – 96.21 = 7.44
Solution (2)

Let the default probability per year = Q.

The recovery rate is flat, 30% of face value.

So if the notional principal is 100, we can recover 30.

We can work out the present value of losses assuming the default may happen at the end of years 1, 2, 3, 4.

Accordingly, we calculate the present value of the risk-free bond at the end of years 1, 2, 3, 4.

Then we subtract 30 being the recovery value each year.

We then calculate the present value of the losses using continuously compounded risk-free rate.
Solution (3)

✿ **PV factors**

✿ $e^{-0.015} = .9851$

✿ $e^{-0.030} = .9704$

✿ $e^{-0.045} = .9560$

✿ $e^{-0.060} = .9417$

✿ $e^{-0.075} = .9277$

✿ $e^{-0.09} = .9139$
Time of default = 1

- PV of risk free bond
  $= 2 + 2e^{-0.015} + 2e^{-0.030} + 2e^{-0.045} + 2e^{-0.060} + 2e^{-0.075} + (102)e^{-0.090}$
  $= 2[1 + .9851 + .9704 + .9560 + .9417 + .9277] + (102)(.9139)$
  $= 11.56 + 93.22 = 104.78$

Time of default = 2

- PV of risk free bond
  $= 2[1 + .9851 + .9704 + .9560] + (102)(.9417) = 103.88$

Time of default = 3

- PV of risk free bond
  $= 2[1 + .9851] + (102)(.9704) = 102.95$

Time of default = 4

- PV of risk free bond = 102
Solution (Cont…)

<table>
<thead>
<tr>
<th>Default point (Years)</th>
<th>Expected losses</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(104.78 – 30)Q = 74.78Q</td>
<td>(74.78)Qe^{-0.03} = 72.57Q</td>
</tr>
<tr>
<td>2</td>
<td>(103.88 – 30)Q = 73.78Q</td>
<td>(73.88)Qe^{-0.06} = 69.58Q</td>
</tr>
<tr>
<td>3</td>
<td>(102.95 – 30)Q = 72.95Q</td>
<td>(72.95)Qe^{-0.09} = 66.67Q</td>
</tr>
<tr>
<td>4</td>
<td>(102.00 – 30)Q = 72.00 Q</td>
<td>(72.00)Qe^{-0.12} = 63.86Q</td>
</tr>
<tr>
<td></td>
<td></td>
<td>272.68 Q</td>
</tr>
</tbody>
</table>

So we can equate the expected losses:

- i.e., \( 7.44 = 272.68Q \)
- or \( Q = 0.0273 = 2.73\% \)
Problem

- A company has issued 3 and 5 year bonds with a coupon of 4% per annum payable annually.

- The continuously compounded yields on the bonds are 4.5% and 4.75% respectively.

- Risk free rate with continuous compounding is 3.5% for all maturities. The coupon for the risk free bond is also 4%.

- The recovery rate is flat 40. Defaults can take place at the middle of the year.

- The risk neutral default rates are Q1 for years 1-3 and Q2 for years 4-5. Find Q1 and Q2.
3 year bond

<table>
<thead>
<tr>
<th>CF</th>
<th>Year</th>
<th>r</th>
<th>PVF</th>
<th>PV</th>
<th>r_f</th>
<th>PVF</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>-0.045</td>
<td>0.955997</td>
<td>3.82399</td>
<td>-0.035</td>
<td>0.965605</td>
<td>3.862422</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>-0.045</td>
<td>0.913931</td>
<td>3.655725</td>
<td>-0.035</td>
<td>0.932394</td>
<td>3.729575</td>
</tr>
<tr>
<td>104</td>
<td>3</td>
<td>-0.045</td>
<td>0.873716</td>
<td>90.86645</td>
<td>-0.035</td>
<td>0.900325</td>
<td>93.63375</td>
</tr>
</tbody>
</table>

\[
\text{PV of expected loss} = 98.34617
\]

<table>
<thead>
<tr>
<th>Time</th>
<th>p</th>
<th>Recovery</th>
<th>Risk free value</th>
<th>LGD</th>
<th>K using risk free rate</th>
<th>PV of expected loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>Q_1</td>
<td>40</td>
<td>103.01</td>
<td>63.01</td>
<td>.9827</td>
<td>61.92 Q_1</td>
</tr>
<tr>
<td>1.5</td>
<td>Q_1</td>
<td>40</td>
<td>102.61</td>
<td>62.61</td>
<td>.9489</td>
<td>59.41 Q_1</td>
</tr>
<tr>
<td>2.5</td>
<td>Q_1</td>
<td>40</td>
<td>102.20</td>
<td>62.20</td>
<td>.9162</td>
<td>56.98 Q_1</td>
</tr>
</tbody>
</table>

\[
178.31 Q_1 = 101.23 - 98.35.
\]

So \( Q_1 = .0161 \)
## 5 year bond

<table>
<thead>
<tr>
<th>CF</th>
<th>Year</th>
<th>r</th>
<th>PVF</th>
<th>PV</th>
<th>rf</th>
<th>PVF</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>-0.0475</td>
<td>0.95361</td>
<td>3.814442</td>
<td>-0.035</td>
<td>0.965605</td>
<td>3.862422</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>-0.0475</td>
<td>0.909373</td>
<td>3.637492</td>
<td>-0.035</td>
<td>0.932394</td>
<td>3.729575</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>-0.0475</td>
<td>0.867188</td>
<td>3.46875</td>
<td>-0.035</td>
<td>0.900325</td>
<td>3.601298</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>-0.0475</td>
<td>0.826959</td>
<td>3.307837</td>
<td>-0.035</td>
<td>0.869358</td>
<td>3.477433</td>
</tr>
<tr>
<td>104</td>
<td>5</td>
<td>-0.0475</td>
<td>0.788597</td>
<td>82.01408</td>
<td>-0.035</td>
<td>0.839457</td>
<td>87.30353</td>
</tr>
</tbody>
</table>

\[
\text{96.2426} + 101.9743 = 180.56 \quad Q_1 + 108.53Q_2
\]

<table>
<thead>
<tr>
<th>Time</th>
<th>p</th>
<th>Recovery</th>
<th>Risk free value</th>
<th>LGD</th>
<th>K using risk free rate</th>
<th>PV of expected loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>Q_1</td>
<td>40</td>
<td>103.77</td>
<td>63.77</td>
<td>0.9827</td>
<td>62.67 Q_1</td>
</tr>
<tr>
<td>1.5</td>
<td>Q_1</td>
<td>40</td>
<td>103.40</td>
<td>63.40</td>
<td>0.9489</td>
<td>60.16 Q_1</td>
</tr>
<tr>
<td>2.5</td>
<td>Q_1</td>
<td>40</td>
<td>103.01</td>
<td>63.01</td>
<td>0.9162</td>
<td>57.73 Q_1</td>
</tr>
<tr>
<td>3.5</td>
<td>Q_2</td>
<td>40</td>
<td>102.61</td>
<td>62.61</td>
<td>0.8847</td>
<td>55.39 Q_2</td>
</tr>
<tr>
<td>4.5</td>
<td>Q_2</td>
<td>40</td>
<td>102.20</td>
<td>62.20</td>
<td>0.8543</td>
<td>53.19 Q_2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>180.56 Q_1 + 108.53Q_2</td>
</tr>
</tbody>
</table>

\[
180.56 Q_1 + 108.53Q_2 = 101.97 - 96.24, \quad \text{So} \quad Q_2 = .0260
\]
Problem

- A bank has made a loan commitment of $2,000,000 to a customer. Of this, $1,200,000 has been disbursed. There is a 1% default probability and 40% loss given default. In case of default, drawdown is expected to be 75%. What is the expected loss?

Solution

- Drawdown in case of default
  \[= (2,000,000 - 1,200,000) \times (.75) = 600,000\]

- Adjusted exposure
  \[= 1,200,000 + 600,000 = 1,800,000\]

- Loss given default
  \[= (.01) \times (.4) \times (1,800,000) = $7,200\]
Credit Loss Distribution

Consider a portfolio of $100 million with three bonds, A, B, and C, with various probabilities of default. Assume exposures are constant, recovery in case of default is zero, and default events are independent across issuers. Construct a credit loss distribution.

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Exposure</th>
<th>Default Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$25</td>
<td>0.05</td>
</tr>
<tr>
<td>B</td>
<td>$30</td>
<td>0.10</td>
</tr>
<tr>
<td>C</td>
<td>$45</td>
<td>0.20</td>
</tr>
</tbody>
</table>
## Portfolio Exposures, Default Risk & Credit Losses

<table>
<thead>
<tr>
<th>Issuer</th>
<th>Exposure</th>
<th>Default Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$25</td>
<td>0.05</td>
</tr>
<tr>
<td>B</td>
<td>$30</td>
<td>0.10</td>
</tr>
<tr>
<td>C</td>
<td>$45</td>
<td>0.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Default</th>
<th>Loss</th>
<th>Probability</th>
<th>Cumulative Probability</th>
<th>Expected Loss</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>$0</td>
<td>0.6840</td>
<td>0.6840</td>
<td>0.000</td>
<td>120.08</td>
</tr>
<tr>
<td>A</td>
<td>$25</td>
<td>0.0360</td>
<td>0.7200</td>
<td>0.900</td>
<td>4.97</td>
</tr>
<tr>
<td>B</td>
<td>$30</td>
<td>0.0760</td>
<td>0.7960</td>
<td>2.280</td>
<td>21.32</td>
</tr>
<tr>
<td>C</td>
<td>$45</td>
<td>0.1710</td>
<td>0.9670</td>
<td>7.695</td>
<td>172.38</td>
</tr>
<tr>
<td>A, B</td>
<td>$55</td>
<td>0.0040</td>
<td>0.9710</td>
<td>0.220</td>
<td>6.97</td>
</tr>
<tr>
<td>A, C</td>
<td>$70</td>
<td>0.0090</td>
<td>0.9800</td>
<td>0.630</td>
<td>28.99</td>
</tr>
<tr>
<td>B, C</td>
<td>$75</td>
<td>0.0190</td>
<td>0.9990</td>
<td>1.425</td>
<td>72.45</td>
</tr>
<tr>
<td>A, B, C</td>
<td>$100</td>
<td>0.0010</td>
<td>1.0000</td>
<td>0.100</td>
<td>7.53</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td></td>
<td>13.25</td>
<td>434.7</td>
</tr>
</tbody>
</table>
Problem

- Suppose a bank has three transactions worth of $10 million, $30 million, and −$25 million. What is the exposure with netting and without netting?

- Without netting, the exposure is \((10 + 30) = 40\) million.

- With netting, the exposure is \((10 + 30 - 25) = 15\) million.
Problem

- A diversified portfolio of OTC derivatives has a gross marked to market value of 4,000,000 and a net value of $1,000,000. If there is no netting agreement in place, calculate the current credit exposure.

- \[ x + y = 4,000,000 \]
- \[ x - y = 1,000,000 \]

- So \( x = 2,500,000 \) and \( y = 1,500,000 \)

- So credit exposure to counterparty = $2,500,000.
Problem

- A bond with a face value of $100,000 has a 40% probability of default with a recovery rate of 50%. The bond is selling for $70,000. Calculate the mean loss rate and the risk neutral mean loss rate.

- Mean loss rate = Expected loss = \( \frac{0.5 \times 0.4 \times 100,000}{100,000} \)
  \[ = 20,000/100000 = 0.2 = 20\% \]

- Risk neutral loss mean rate = \( \frac{100,000 - 70,000}{100,000} \)
  \[ = 30\% \]
Problem

- A bank makes a $100,000,000 loan at a fixed interest rate of 8.5% per annum.
- The cost of funds for the bank is 6.0%, while the operating cost is $800,000.
- The economic capital needed to support the loan is $8 million which is invested in risk free instruments at 2.8%.
- The expected loss for the loan is 15 basis points per year.
- What is the risk adjusted return on capital?

Net profit = $100,000,000 \times (0.085 - 0.060 - 0.0015) - 800,000 + (8,000,000) \times 0.028

= 23,50,000 - 800,000 + 224,000

= $1,774,000

Risk adjusted return on capital = \frac{1.774}{8} = 0.22175 = 22.175\%